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## Train Scheduling in High Speed Railways: Considering Competitive Effects

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### Abstract

The railway planning problem is usually studied from two different points of view: macroscopic and microscopic. We propose a macroscopic approach for the high-speed rail scheduling problem where competitive effects are introduced. We study train frequency planning, timetable planning and rolling stock assignment problems and model the problem as a multi-commodity network flow problem considering competitive transport markets. The aim of the presented model is to maximize the total operator profit.

We solve the optimization model using realistic problem instances obtained from the network of the Spanish railway operator RENFE, including other transport modes in Spain.

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### 1. Introduction

High-Speed Railways (HSR) are currently regarded as one of the most significant technological breakthroughs in passenger transport developed in the second half of the 20th century. This type of rail network is devoted to providing high-speed services to passengers willing to pay for shorter travel time and a quality improvement in rail transport. Since the earliest projects started commercial operation in the 1970s, high-speed rail has been presented as

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a success story in terms of demand and revenues. It has particularly been viewed in many countries as a key factor for the revival of railway passenger traffic, a declining business that had lost its momentum due to the fierce competition of road and air transport.

However, building, maintaining and operating HSR lines is expensive, involves a significant amount of resources and may substantially compromise both the transport policy of a country and the development of its transport sector for decades. Because of the upcoming deregulation and the advent of high-speed rail networks, European passenger railroads are battling for customers among themselves and with other means of transportation. To maintain a competitive advantage, they rely on the scheduling process as a key factor in winning market share.

Like airlines, rail operators today are looking for advanced decision-support tools in the areas of pricing, yield management, schedule planning, and control. Scheduling, with its product-positioning component, is one of the first steps in the planning process. The railway operator identifies needs of the customer, and once the route structure of the railroad is determined, designs the operating schedule that represents the services to be offered. Scheduling is therefore a fundamental component in maximizing overall profit.

Nowadays, scheduling has become more complex. This complexity stems from the operators' need to build schedules to fit a changing demand, to meet both constraint-driven and market-driven criteria, and to allow adjustments. Demand models are used to develop forecasts of passenger demand for each origin-destination pair as a function of attributes such as average fares, frequencies, market demographics and economic conditions. Given these total demand estimates, passenger choice models are used to estimate for each competitor and each market, the proportion of demand (or share) it captures in that market, considering market-specific characteristics, including passengers' mode preferences, fares, frequencies and other characteristics (Ben-Akiva and Lerman, 1985).

For tractability purposes, schedule design models typically consider demand for services to be deterministic and invariant to schedule changes and competition. These assumptions, however, have been shown to lead to overestimates of the number of passengers served, the revenue captured, and schedule profitability (Belobaba, 2009). An effective schedule planning process, then, depends critically on both the accurate estimation of the overall demand for travel; and the accurate understanding of how passengers will choose between the competitors' travel options.

The motivation for considering multi-modal competition stems from the fact that HSRs and airlines are increasingly competing for passengers in many parts of Europe and Asia, especially in short- to medium-haul markets. The HSR often competes by providing similar or even greater service frequency and better connectivity to the city centers. Moreover, the HSR is often perceived as the safest and most comfortable mode (Jehanno et al., 2011).

## **2. State of the art**

Cordeau et al. (1998) present a survey of optimization models for train routing and scheduling problems. Within the scheduling process, train timetabling (Caprara et al., 2002) and rolling stock assignment (Cadarso and Marín, 2011) may be studied from a macroscopic point of view. Because detailed infrastructure knowledge is not available (within a macroscopic approach), high-level planning alone may result in conflicts, which would hinder the solutions from being operable. However, incorporating detailed infrastructure information will lead to an extremely huge-sized optimization problem. Therefore, a two-phase approach is usually preferred as a compromise.

The first phase is usually the macroscopic one, where the problem is usually modelled as a multi-commodity network flow problem considering the train unit scheduling (Ghoseiri et al., 2004; Cacchiani et al., 2010; Cadarso, 2013; Cadarso and Marín, 2010, 2011, 2012 and 2014). In this phase, the train sequence and rolling stock assignment problems are studied. Real world constraints are considered jointly with an objective function which minimizes the total operating cost based on train service and empty movement operating costs, routing preferences and penalties for train and infrastructure shortages and so on.

In the second phase, operational plans are usually studied in detail. Real world scenarios are considered at the microscopic modelling level, i.e. compatibility issues, time allowances for coupling/decoupling activities, and conflicts at the system are studied (Kroon et al., 2008). The train scheduling during real-time is studied in order to obtain a conflict-free timetable, considering a careful estimation of time separation among trains (D'Ariano et al., 2007; Espinosa-Aranda and García-Ródenas, 2013).

### 2.1 Contributions

Our work differs from others in that we develop a new approach that generates HSR schedules using an integrated schedule optimization model that accounts for passenger demand use capturing the impacts of schedule decisions on passenger demand. The main contributions of this paper are summarized as follows:

1. We develop a macroscopic integrated scheduling model that includes frequency planning, approximate timetable planning, rolling stock assignment and passenger demand served.
2. We address passenger demand choice through a logit function: the demand split across modes depends on the service frequency (among others), which is a model variable;
3. We carry out computational experiments on realistic study cases of the Spanish rail operator RENFE.

### 2.2 Outline of the paper

This paper is organized as follows. Section 3 introduces our demand modelling approach. Section 4 describes the problem in detail. Section 5 is devoted to the mathematical model. In Section 6 some computational experiments are presented. Finally, we draw some conclusions.

## 3. Demand modeling

Service frequency is the most important attribute on which transport operators compete. They can attract more passengers in a market by increasing the frequency on its Origin-Destination (OD) pair. For a given unconstrained total demand, the market share of each operator depends, among other factors, on its own frequency and on the frequency of its competitors.

However, modelling the market share as simply a function of the frequency share is not enough to model passenger demand behaviour in many markets. This is especially true in markets where the competitor fares are different from each other. There are other attributes, such as fares and travel times that can significantly affect passengers' choice. Many past studies have modelled market share as a function of the attributes (i.e., frequency, price and travel time) (Wei and Hansen, 2005; Vaze and Barnhart, 2012a). A standard discrete choice approach is based on a logit model where the choice probability is proportional to the exponentials of the systematic utilities of each operator  $o$  for market  $w$  ( $v(o|w)$ ),

$$P_o^w = \frac{e^{v(o|w)}}{\sum_{o' \in O} e^{v(o'|w)}}, \quad o \in \{HSR, others\} \quad (1)$$

, where  $f_o^w$  is the service frequency of operator  $o$  in market  $w$ ,  $p_o^w$  is the price of operator  $o$  in market  $w$ ,  $tt_o^w$  is the planned travel time of operator  $o$  in market  $w$ ,  $\alpha$  is the frequency parameter,  $\beta$  is the price parameter and  $\gamma$  is the trip time parameter.  $O$  is the operator set.

The captured demand by operator  $o$  depends on competition effects, as explained before. Consequently, unconstrained demand gets split between different operators. The captured demand by operator  $o$  in market  $w$  is  $P_o^w d_w$ , where  $d_w$  is the unconstrained demand in market  $w$ . As we are studying a tactical problem for which the timetable is approximate, we model captured demand as a function of the total frequency (i.e., across the planning period) offered in the OD pair; i.e., the OD attributes are substituted in the logit model for the frequency parameter (i.e.,  $f_o^w$  is substituted by  $f_o^{od}$  being  $od$  the OD pair of market  $w$ :  $v(o|w) = \alpha f_o^{od} + \beta p_o^w + \gamma tt_o^w$ ).

## 4. Problem description

In this section, the train scheduling problem in high-speed rail networks is described in detail. First, the railway infrastructure is introduced. Next, we describe train services timetable and rolling stock. Finally, we explain how we treat the passenger demand.

#### 4.1. Railway infrastructure

The railway network consists of tracks and stations. We model the infrastructure as a graph with nodes representing the stations, and with directed arcs (the existing infrastructure linking different stations is represented by arcs). Between two stations, two different arcs exist, one for each direction of movement. Therefore, every arc is defined by its departure and arrival station and by its length (e.g., in kilometers). Depot stations form a subset of the stations; these are the locations where trains are parked or shunted.

The planning time is discretized into time periods. We study a tactical problem where a schedule is to be determined for a planning horizon of, say, one week. In order to ensure tractability of the problem, we propose an aggregated network, similar to that presented in Harsha (2008). It allows different levels of time-discretization at each station. Such an aggregation scheme reduces problem size, without compromising the modelling accuracy too much.

#### 4.2. Timetable and rolling stock

The train services are grouped in lines. A line is characterized by its terminal stations, by a path through the infrastructure between the terminals, and by a set of stations along the path. Train services run up and down between the terminals and call at the specified stations underway. We assume that the timetable departure times and frequencies are publicly available, so the passengers know when the trains depart and plan their traveling accordingly. Finally, we assume that the train schedule will be periodic, that is, the schedule will repeat after the planning period ends. To satisfy this, the number of train units of each type at each station at the beginning and the end of the planning period must be the same.

There are self-propelled train units of different types; they all have a driver seat at both ends. Units of the same type can be attached to each other to form trains compositions. A composition of train units is a sequence of elements of the same type. Each line is served with one train unit type. Each train unit type has a given capacity; this value includes seated passengers. The capacity imposes a hard limit of how many passengers fit into the train.

#### 4.3. Passenger demand

The demand is characterized by an origin, a destination and a departure time. This information may be represented by market  $w$ , defined by a departure station, an arrival station, and the desired departure time. The size of the market is denoted by  $g_w$ . In order to calculate the demand split between the different travel options, we use the expression in (1).

An important modeling issue is how to capture the passenger demand satisfaction requirements for every market (Vaze and Barnhart, 2012b). In each market, passengers can choose any of the corresponding itineraries. Thus the proposed model is itinerary-based. The demand captured by the train operator will depend on competition effects which are incorporated in our model through the logit model introduced in Section 3.

### 5. High-speed railway scheduling model

The aim of the Integrated High-Speed Railway Scheduling Model (IHSRASM) is to maximise operator's profit, while passenger demand requirements are satisfied and operator's operational limitations matched.

Within the scheduling process, train timetabling and rolling stock assignment may be studied from a macroscopic point of view. However, shunting operations based on detailed infrastructure knowledge belong to a lower level planning. Therefore, high-level planning alone may result in conflicts such as crossing, which would hinder the solutions from being operable. However, incorporating detailed infrastructure information into our scheduling problem will lead to an extremely huge-sized optimization problem instance intractable to solve. Therefore, a two-phase approach is preferred as a compromise: a macroscopic and a microscopic approach. The microscopic approach is presented in Espinosa-Aranda et al. (2014).

In our model, the relationships between the data and variables are considered within a directed space–time graph,  $G(S,A)$ , where  $S$  is the set of stations and  $A$  is the set of arcs. The purpose of the constraints is summarized as follows:

1. As for the schedule, service frequency is determined, approximate departure times are decided and headway times are enforced.
2. The passenger demand, which depends on the variable of service frequency, is linked to the capacity of the allocated train units.
3. As for the rolling stock, the amount of used rolling stock is limited; each trip gets a composition assigned; the storage of the stations is controlled.

### 5.1. Sets

In order to be able to formulate the IHSRASM, we need to define the following sets:

$S$  : set of nodes indexed by  $s$ ; each node represents a station.

$DS$  : set of depot stations indexed by  $s$ .

$OD$  : set of origin-destination pairs indexed by  $od$ .

$W$  : set of markets indexed by  $w$ ; each element in this set is defined by an origin-destination pair and a departure time period.

$A$  : set of tracks indexed by  $a$ ; each element is defined by (consecutive) origin and destination stations.

$C$  : set of train compositions indexed by  $c$ .

$L$  : set of train services indexed by  $l$ ; each element is defined by origin and destination stations, a departure time period, an arrival time period and a route.

$\Xi$  : set of lines indexed by  $\xi$ ; each element is defined by a sorted set of tracks.

$I$  : set of itineraries indexed by  $i$ ; each element is defined by origin and destination stations, the tracks which connect them, and the departure time.

$I_w$  : subset of itineraries serving market  $w$ .

$I_l$  : subset of itineraries using train service  $l$ .

$I_a$  : subset of itineraries using arc  $a$ .

$L_{od}$  : subset of services attending the pair of stations  $od$ .

$L_a$  : subset of train service using arc  $a$ .

$L_\xi$  : subset of train service using line  $\xi$ .

$PAS_{s,t}$  : subset of train services coming through station  $s$  at period  $t$ .

### 5.2. Parameters

$p_i^w$  : average ticket price for a passenger in market  $w$  travelling in itinerary  $i$ .

$\pi_l^c$  : unitary operating cost for train service  $l$  with composition  $c$ .

$g_w$  : size of market  $w$ .

$a_{l,s,t}$  : timetable of train service  $l$ ; it takes value 1(-1) if service  $l$  arrives at (departs from) station  $s$  during time period  $t$ .

$q_c$  : seating capacity in composition  $c$ .

$n_c$  : fleet size for composition  $c$ .

$P_{HSR}^w$  : probability of passengers from market  $w$  selecting alternative HSR among all the alternatives. Note that this depends on the frequency decision variable.

$\tau_l$  : running time for train service  $l$ .

$b_c$  : average available running time across the planning period per composition  $c$ .

$h_{s,t}$  : average headway capacity for station  $s$  during time period  $t$ .

$tu_c$  : number of train units in composition  $c$ .

$cap_s$  : train unit capacity for each station  $s$ .

### 5.3. Variables

The most central decision variables are  $k_l^c \in \mathbb{Z}^+$ , defined for  $l \in L, c \in C$ . Their values indicate the frequency number for a service  $l \in L$  with composition  $c \in C$ . The model contains the following additional variables:

$f_{od}$  : service frequency across the planning period in  $od$  pair.

$g_i^{w,\xi}$  : number of passengers from market  $w$  that travel in itinerary  $i$  within line  $\xi$ .

$y_s^c$  : train inventory of composition  $c$  in station  $s$  at the beginning of the planning period.

### 5.4. Objective function

The objective function of the model reads as follows:

$$\max z_{HSR} = \sum_{w \in W} \sum_{i \in I_w} \sum_{\xi \in \Xi} p_i^w g_i^{w,\xi} - \sum_{l \in L} \sum_{c \in C} \pi_l^c k_l^c \quad (2)$$

The objective terms, in the given order, model the following quantities: revenue given by served passengers and operating costs of scheduled train services.

### 5.5. Scheduling constraints

The first set of scheduling constraints enforces the headway requirements.

$$\sum_{l \in PAS_{s,t}} \sum_{c \in C} k_l^c \leq h_{s,t} \quad \forall s \in S, \forall t \in T \quad (3)$$

$$f_{od} \leq \sum_{l \in L_{od}} \sum_{c \in C} k_l^c \quad \forall od \in OD \quad (4)$$

$$\sum_{t \in T} \sum_{l \in L: d_{l,s,t} = -1} k_l^c = \sum_{t \in T} \sum_{l \in L: d_{l,s,t} = +1} k_l^c \quad \forall s \in DS, c \in C \quad (5)$$

Constraints (3) are headway requirements in term of station and time period. Constraints (4) make sure that the service frequency in each  $od$  pair is not greater than the number of operated train services. Constraints (5) ensure that the operated schedule is symmetric, i.e., the number of departures and arrivals at each station across the planning period is the same.

### 5.6. Passenger demand constraints

$$\sum_{i \in I_w} \sum_{\xi \in \Xi} g_i^{w,\xi} \leq P_{HSR}^w g_w, \quad \forall w \in W \quad (6)$$

$$\sum_{i \in I_l \cap I_a} \sum_{w \in W} g_i^{w,\xi} \leq \sum_{c \in C} q_c k_l^c \quad \forall a \in A, \xi \in \Xi, l \in L_a \cap L_\xi \quad (7)$$

Constraints (6) ensure that the number of passengers served in each market does not exceed those captured in that market. Note that  $P_{HSR}^w$  depends on the decision variables  $f_{od}$ . The constraints (7) say that for each arc  $a \in A$ , line  $\xi \in \Xi$  and train service  $l \in L$ , the combined capacity of the trains on the arc during the time interval is enough to accommodate the passenger demand.

### 5.7. Rolling stock constraints

$$\sum_{s \in S} y_s^c \leq n_c \quad \forall c \in C \quad (8)$$

$$\sum_{l \in L} \tau_l k_l^c \leq b_c n_c \quad \forall c \in C \quad (9)$$

$$y_s^c \geq \sum_{t' \in T: t' \leq t} \left( \sum_{l \in L: a_{l,s,t'} = -1} k_l^c - \sum_{l \in L: a_{l,s,t} = 1} k_l^c \right) \quad \forall s \in DS, t \in T, c \in C \quad (10)$$

$$\sum_{c \in C} t_{c,s} y_s^c \leq cap_s \quad \forall s \in S \quad (11)$$

The constraints (8) are fleet size constraints. Fleet utilization constraints (9) ensure that the utilization of each composition is not greater than the available total block hours during the planning period. The constraints (10) are flow balance constraints at the depot stations. Constraints (11) are the depot train capacity at station  $s$ .

### 5.8. Variable definition

$$k_l^c \in \mathbb{Z}^+ \quad \forall l \in L, c \in C \quad (12)$$

$$f_{od} \in \mathbb{Z}^+ \quad \forall od \in OD \quad (13)$$

$$y_s^c \in \mathbb{Z}^+ \quad \forall s \in S, \forall c \in C \quad (14)$$

$$g_i^{w,\xi} \in \mathbb{R}^+ \quad \forall w \in W, \forall i \in I_w, \forall \xi \in \Xi \quad (15)$$

The constraints (12)-(15) are variable domain constraints.

## 6. Computational experiments

Our experiments are based on realistic cases drawn from RENFE's Madrid-Seville high speed rail corridor in Spain (Figure 1). This corridor consists of three types of lines. Lines 2 and 3 are operated with the same rolling stock, which differs from the one used in line 1 (see Table 1). Figure 1 shows in detail the stations, railway segments and the definition of the 3 lines. Table 2 shows operating costs for the train services; note that every train service within a line operates all the line; consequently, there are four different operating costs (recall there are two types of rolling stock). Table 3 shows the size of each market within a day. Tables 4 and 5 show the average ticket prices for line 1 and lines 2 and 3, respectively. A weekly planning is considered, with a daily time interval between [4; 24] hours.

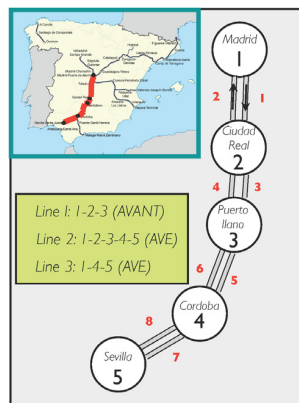


Figure 1: Madrid-Seville corridor

Table 1. Capacities for the different rolling stock types

Composition	Line 1	Line 2	Line 3
1	237	308	308
2	474	616	616

Table 2. Operating costs

Composition	Line 1	Line 2	Line 3
1	3837.6	9415	9415
2	5756.4	14122.5	14122.5

Table 3. Daily market sizes

OD	1	2	3	4	5
1	-	1377	479	185	3270
2	1377	-	216	185	185
3	479	216	-	185	185
4	185	185	185	-	1291
5	3270	185	185	1291	-

Table 4. Average ticket prices in line 1

OD	1	2	3	4	5
1	-	26.2	32.3	-	-
2	26.2	-	6.7	-	-
3	32.3	6.7	-	-	-
4	-	-	-	-	-
5	-	-	-	-	-

Table 5. Average ticket prices in lines 2 and 3

OD	1	2	3	4	5
1	-	36.4	45.7	62.1	75.5
2	36.4	-	17.3	38.6	57.2
3	45.7	17.3	-	30.1	52.6
4	62.1	38.6	30.1	-	30.1
5	75.5	57.2	52.6	30.1	-

The IHSRASM is a non-linear mixed integer programming model. The non-linearity is in constraints (6). The captured demand is a non-linear function of the train operator's frequency values ( $f_{od}$ ). In order to solve the model, we linearize this expression using piecewise functions. We approximate the relationship between the fraction of passengers selecting the HSR and the frequency values by a piecewise linear function. In order to linearize the non-linear relationship we use special ordered set variables, which is an ordered set of non-negative variables, of which at most two can be non-zero, and if two are non-zero these must be consecutive in their ordering. Special ordered



sets are typically used to approximately incorporate non-linear functions of a variable into a linear model. When embedded in a Branch and Bound code these variables enable truly global optima to be found, and not just local optima (Beale and Tomlin, 1970). We used for our tests a personal computer with an Intel Core i7 at 2.8 GHz and 8 GB of RAM, running under Windows 7 64-Bit, and we implemented the models in GAMS/Cplex 12.1. We solved all models to a maximum 1% optimality gap.

Our case study focuses on studying the sensibility of the solutions to the competitors' frequency. Table 6 shows these results. The first column (COM) shows the competitors' frequency value. The second column (FREQ) shows the frequency value as predicted by the IHSRASM. The third column (%MS) displays the average market share the train operator will be able to serve with the predicted schedule. The fourth column (RSD) shows the rolling stock distribution by depot station at the beginning of the planning period; this column is subdivided into three different sub columns, one per depot station (i.e., Madrid (MAD), Puertollano (PU) and Seville (SEV)), and again subdivided into two different sub columns, one per available train composition (i.e., c1, c2). The fifth column shows the objective function value (OF) for each study case. Finally, the last two columns show the number of iterations (#ITER) and computational time (TIME) needed to reach the solution.

When the frequency of the competitors increases, the market share obtained by the train operator strongly decreases. However, the frequency value as predicted by the IHSRASM slightly decreases because the train operator tries to maintain its competitive presence in the markets. It is also useful to know the optimal initial distribution of the rolling stock, which varies depending on the schedule to be implemented. Obviously, when the market share drops, the number of served passengers also drops, and therefore the profit, which is given by the objective function, strongly drops.

Table 6. IHSRASM solutions

COM	FREQ	%MS	RSD						OF(x10 <sup>6</sup> )	#ITER	TIME(s)
			MAD		PU		SEV				
			c1	c2	c1	c2	c1	c2			
70	331	0,85	6	5	0	0	4	5	12,5	17859	491,75
100	320	0,79	6	4	0	1	4	5	8,33	325846	254,87
150	317	0,70	3	5	0	1	7	4	8,05	10438314	6466
200*	317	0,63	4	4	0	1	6	5	7,54	1277000	1325
500	324	0,38	1	5	2	0	7	5	3,74	2075	162,85

\*this case study was solved to a 3.5% optimality gap

## Conclusions

We have developed an integrated scheduling and rolling stock model in order to study the high-speed rail tactical planning. The model includes frequency planning, approximate timetable, rolling stock assignment and passenger demand choice. We formulate the model using mixed integer non-linear programming. The novel aspects of this formulation are the modelling of the modal and operator choice by the passengers, that is, the unconstrained demand is split between the different operators. The captured demand is modeled as a function of the total weekly frequency offered in the OD pairs. The model is solved using real data instances obtained from RENFE, the Spanish high-speed rail operator.

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